Chapter 9

SPATIAL VARIABILITY

This chapter will examine the spatial variability of evaporation and some soil properties associated with evaporation, namely the soil surface temperature and profile water content. The change in storage due to irrigation will also be examined. Data sets that will be used are those that were used in Chapter 8; midday temperature difference between dry and drying soil, measured in Experiments 2 and 3; profile water contents and change in storage due to irrigation, measured in Experiment 2; and evaporation from ML's measured in Experiment 3. Each of these variables was measured at the 57 field locations described in Chapter 2.

Each variable was also measured at several different times following irrigation, usually on a daily basis. The temporal nature of these data sets complicates the analysis of spatial variability since the nature of this variability may change with time. However, as pointed out in the previous chapter, time invariance of a data set implies at least partial time invariance of the spatial variability. This chapter will demonstrate one method, the relative variogram, of combining data from several days into a single expression of the spatial variability.

Several researchers have studied the spatial variability of soil water content and surface temperature (e.g. Vauclin et al. 1982, Viera et al. 1983, Yates and Warrick 1987, Mulla 1988, Yates et al. 1988). Among tools used by these and other authors to describe spatial variability are the covariance, the autocorrelation function, the variogram and the covariogram. Use of these tools implies the hypothesis that sample values are not randomly distributed in space but are to some degree autocorrelated in space with samples taken closer together more likely to have similar values than those taken farther apart. Two reviews of spatial variability analysis, with examples pertaining to soil and water science, are by Vieira et al. (1983) and Warrick et al. (1986). Unless otherwise noted the following discussion derives from these two sources.

The Variogram.

The most useful tool for studying spatial variability may be the variogram since it can be used in the kriging process to estimate values of the variable at unsampled locations in the field. The variogram, $\gamma(h)$, is defined by:

$$\gamma(h) = \mathrm{Var}[Z(x) - Z(x + h)]/2 \qquad [9\text{-}1]$$

where Var is the variance operator. The location vector, x, represents the position of each sample value, Z(x), in 1, 2 or

3 dimensions. The separation vector, h, represents the distance and direction between sample pairs. The variogram is defined under the intrinsic hypothesis: 1) the expected value of Z(x) exists and has the same value over all subparts of the region of interest (no drift); 2) Var[Z(x) - Z(x + h)] is defined for all vectors, h, and is a unique function of h.

The variogram may be estimated by the sample variogram, γ^* which is:

$$\gamma^*(h) = 1/[2\ N(h)] \sum_{i=1}^{N(h)} [Z(x_i + h) - (Z(x_i)]^2 \qquad [9\text{-}2]$$

where N(h) is the number of sample pairs separated by the vector h, and the x_i are the samples so separated. In practice the vector h is defined as a class with upper and lower limits on the direction and distance. This has the advantage of increasing the precision of each estimate by increasing the number of sample pairs included in it. A disadvantage is that as class size increases the distance, h, to which $\gamma^*(h)$ pertains, becomes increasingly ill defined.

Usually the average separation distance, for all data pairs in a class, is used for h in plots of $\gamma^*(h)$ vs. h for example. In practice, as class size changes the sample variogram may change radically. Therefore it is well to calculate $\gamma^*(h)$ for a number of different class sizes in order to choose that class size which is most appropriate (results in variograms which look best to the user). Distance classes are often assigned in a regularly increasing fashion such that each class has identical width. With some data sets this practice has the disadvantage of causing some classes to contain low numbers of data pairs causing γ^* to be poorly estimated. It often happens that the lower numbers of data pairs occur for classes at the low and high ends of separation distance. For use with kriging, information on the shape of the variogram is most important at the lower separation distances. One way to address this problem is to assign class widths dynamically such that each class contains an equal number of data pairs. A disadvantage of this practice is that it may result in some classes being very wide.

The Relative Variogram.

It sometimes happens that stationarity of the mean exists only over discrete subregions of a larger region. In this case the sample variogram may be estimated by Equation 9-2 for each subregion but not for the larger region as a whole. However, variograms from each subregion may be transformed by dividing by the square of the mean resulting in the relative variogram which is defined for the kth region as:

$$\gamma_k(h) = Var[Z(x) - Z(x + h)]/(2\mu_k^2) \qquad [9\text{-}3]$$

where μ_k is the mean for the kth region (Cressie 1985). The sample relative variogram, $\gamma_k^*(h)$, is then:

$$\gamma_k^*(h) = 1/[2\mu_k^2 N(h)] \sum_{i=1}^{N(h)} [Z(x_i + h) - (Z(x_i)]^2 \qquad [9\text{-}4]$$

Sample relative variograms from each region may then be compared and/or combined perhaps resulting in a more precise estimate of the basic variogram shape than would be possible with the more limited number of samples occurring in any one region. The subregions discussed here need not occur in space but may also occur in time. The variables to be examined in this chapter have the common characteristic that separate data sets are available for several different days and the mean for each day is different. Thus the relative variogram may be a useful tool for reducing these data to a single variogram and perhaps improving the variogram estimation.

Kriging.

In order to use the sample variogram information in kriging it is necessary to model the sample variogram. The negative of a variogram model must be a positive definite function. Examples of models fitting this criterion are given by Warrick et al. (1986). Given the sample values $Z(x_i)$ and a variogram model it is possible to estimate the sample value at an unmeasured location, x_o, using the kriging estimator:

$$Z^*(x_o) = \sum_{i=1}^{N} \lambda_i Z(x_i) \qquad [9\text{-}5]$$

where the λ_i are constrained by:

$$\sum_{i=1}^{N} \lambda_i = 1 \qquad [9\text{-}6]$$

Equation 9-6 is equivalent to stating that the expected value of the error of estimation, $[Z^*(x_o) - Z(x_o)]$, is zero. The kriging equations result from minimizing the variance of the error of estimation subject to 9-6 with the result that Equation 9-5 is a best estimator of $Z(x_o)$.

If second order stationarity exists then not only is the mean of the error term zero, but the covariance, $Cov[Z(x),Z(x + h)]$, exists and is uniquely defined for the separation distance, h. This is a stronger assumption than the intrinsic hypothesis but under second order stationarity the kriging variance, $\sigma_E^2(x_o)$, is:

$$\sigma_E^2(x_o) = \sigma^2 - \mu - \sum_{i=1}^{N} \lambda_i Cov(x_o,x_i) \qquad [9\text{-}7]$$

where μ is a Lagrange multiplier which is estimated along with the λ_i by the kriging equations, and σ^2 is the variance of the variable measured. Under second order stationarity the covariance, $Cov(x_o,x_i)$, is:

$$\mathrm{Cov}(x_o, x_i) = \mathrm{Cov}(0) - \gamma(x_i, x_o) \qquad [9\text{-}8]$$

where C(0) is simply the variance of the measured variable and can be estimated by the sample variance. Thus if second order stationarity exists the covariance may be used in place of the variogram in the kriging equations. This results in computational advantages for the matrix inversion.

Cokriging.

Cokriging is a form of kriging using samples from two (or more) variables, say Z_1 and Z_2, measured at the same location. One of the variables may be undersampled with respect to the other so that at every location there is a sample for Z_1 but Z_2 is sampled at only some of the locations. If the variables are correlated cokriging may be used to estimate values of the undersampled variable at locations where the other variable was measured. Regardless of whether one variable is undersampled, the estimation variance of both variables will be reduced by cokriging.

Yates and Warrick (1987), using cokriging of soil moisture content and surface temperature, found that cokriging was worthwhile if the correlation coefficient exceeded 0.5. Mulla (1988) found that cokriging of water content and surface temperature reduced the cokriging estimation variance by one half as compared to the kriging estimation variance for water content. In the present study the midday surface temperature difference between dry and drying soil was found to be highly correlated with evaporation on a daily basis. Therefore it may be possible to use cokriging to improve the estimation of evaporation and/or reduce the number of ML based evaporation samples needed while continuing to sample surface temperature at all locations. Vauclin et al. (1983) and Viera et al. (1982) present the cokriging equations in some detail.

<u>Software</u>.

Sample variogram calculations were done using a program written by the author after that presented by Journel and Huijbregts (1978) and following suggestions by Scott R. Yates. This program allowed class sizes to be dynamically changed so that each class contained equal numbers of sample pairs. This technique facilitated variogram estimation by ensuring that the lowest class contained sufficient pairs to be somewhat independent of extreme values. The program also allowed calculation of the sample relative variogram.

After the sample variograms were estimated and modeled, kriging was done using the GEO-EAS software developed by the U.S. Environmental Protection Agency (Englund and Sparks 1988). This software uses the covariance function in the kriging equations.

Although GEO-EAS allows calculation of the sample relative variogram it provides no means of exporting the calculated values thus frustrating any attempt to combine variograms from different regions (or days). GEO-EAS also calculates the relative variogram differently from Equation 9-4. Rather than dividing each variogam value by the overall mean squared, GEO-EAS calculates the mean only for those sample values used in a given class and divides the sample variogram for each class by the mean squared for that class. Since this study was more concerned with changes in the mean from day to day than from class to class the method shown in Equation 9-4 was preferred.

<u>Statistical Distribution Tests</u>.

If the variable being studied is log-normally distributed interpretation of the sample variogram may be easier if the data are transformed (by taking the natural logarithm) before computing the sample variogram. The Kolmogorov-Smirnov test was used to find if the data were distributed significantly differently than normally or log-normally (Press et al. 1986). The null and alternate hypotheses are:

H_0: The variable has the specified distribution.

H_1: The distribution is other than specified.

The decision rule is:

Accept H_0 if $D \leq c$

where the statistic D is the largest of all absolute values of the differences between the specified CDF and the observed cumulative histogram, and c is the critical value of the statistic for a chosen level of probability. The observed value of the statistic is denoted by 'd'.

In the following tables large values of the observed value, d, and small values of probability indicate that the sample is not normally (or not lognormally) distributed. The probability values are for Prob $(D > d)$. If the null hypothesis is true then the observed value, d, should be smaller than the KS statistic, D. If the probability of this is small then the null hypothesis cannot be accepted (at whatever level of significance is desired).

Table 9-1.

Kolmogorov-Smirnov tests for catch can depths at access tubes, 3 irrigations, Experiment 2.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

	Normal		Log-Normal			c for
Irrig.	KS d	Prob. D>d	KS d	Prob. D>d	N	P(0.10, N)
1	0.1850	0.0404	0.2472	0.0019	57	0.1616
2	0.2309	0.0046	0.3023	0.0001	57	0.1616
3	0.1631	0.0963	0.2264	0.0058	57	0.1616

Catch can depths measured at the 57 access tube locations were neither normally nor log-normally distributed when tested at the 10% level of significance (Table 9-1). However probabilities were much higher, approaching 10%, for the test of normality so for practical purposes these data were considered to be normally distributed.

The change in storage due to irrigation was also neither normally nor log-normally distributed. These tests were significant at the 0.1% level of probability or better (Table 9-2).

Table 9-2.

Kolmogorov-Smirnov tests for change in storage due to irrigation, 3 irrigations, Experiment 2.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

Tests for Normality.

Irrig.	KS d	Prob. D>d	N	c for P(0.10, N)
1	0.3456	0.0000	56	0.1630
2	0.3417	0.0000	56	0.1630
3	0.6938	0.0000	52	0.1692

Tests for Log-Normality.

Irrig.	KS d	Prob. D>d	N	c for P(0.10, N)
2	0.2646	0.0008	56	0.1630
2	0.2884	0.0003	53	0.1676
3	0.8934	0.0000	52	0.1692

Profile water content was measured at the 57 locations on 17 different days. Only on the last day, day 105, was profile water content shown to be distributed other than normally, and this test had a probability of 0.096, just below the 10% level (Table 9-3). On 5 days the data were significantly different from log-normally distributed and on other days probability levels were substantially lower than for the normality tests. Thus these data were assumed to be normally distributed.

The midday temperature depression ($T_{o,max}$ - $T_{d,max}$) was measured on 20 days during Experiment 2. Distributions were significantly different from normal on each of the 3 days after irrigation (days 80, 92 and 100) and also on days 96, 100, 103 and 104 (Table 9-4). Tests for log-normality showed

Table 9-3.

Kolmogorov-Smirnov tests for profile water contents, Experiment 2.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

Tests for Normality.

Day	KS d	Prob. D>d	c for P(0.10, N)	N
77	0.1267	0.3196	0.1616	57
80	0.0963	0.6658	0.1616	57
81	0.1065	0.5489	0.1630	56
82	0.1042	0.5664	0.1616	57
83	0.1008	0.6085	0.1616	57
85	0.1332	0.2638	0.1616	57
90	0.1284	0.3041	0.1616	57
92	0.1297	0.2926	0.1616	57
93	0.0949	0.7053	0.1645	55
94	0.1337	0.2601	0.1616	57
95	0.1305	0.2859	0.1616	57
96	0.1221	0.3635	0.1616	57
98	0.1375	0.2403	0.1630	56
100	0.1439	0.2046	0.1645	55
102	0.1321	0.2928	0.1645	55
103	0.1284	0.3463	0.1676	53
105	0.1708	0.0963	0.1692	52

Tests for Log-Normality.

Day	KS d	Prob. D>d	c for P(0.10, N)	N
77	0.1655	0.0880	0.1616	57
80	0.1247	0.3377	0.1616	57
81	0.1323	0.2805	0.1630	56
82	0.1321	0.2729	0.1616	57
83	0.1287	0.3013	0.1616	57
85	0.1658	0.0871	0.1616	57
90	0.1584	0.1146	0.1616	57
92	0.1569	0.1208	0.1616	57
93	0.1245	0.3617	0.1645	55
94	0.1629	0.0971	0.1616	57
95	0.1604	0.1066	0.1616	57
96	0.1526	0.1406	0.1616	57
98	0.1672	0.0873	0.1630	56
100	0.1647	0.1013	0.1645	55
102	0.1526	0.1541	0.1645	55
103	0.1525	0.1699	0.1676	53
105	0.1904	0.0460	0.1692	52

Table 9-4.

Kolmogorov-Smirnov tests for midday temperature depression ($T_{o,max} - T_{d,max}$), Experiment 2.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

	Tests for Normality.				Tests for Log-Normality.			
Day	KS d	Prob. D>d	c for P(0.10, N)	N	KS d	Prob. D>d	c for P(0.10, N)	N
80	0.8159	0.0000	0.1660	54	0.8148	0.0000	0.1660	54
81	0.1646	0.0962	0.1630	56	0.1880	0.0382	0.1630	56
82	0.0851	0.8035	0.1616	57	0.0910	0.7332	0.1616	57
83	0.0629	0.9779	0.1616	57	0.0811	0.8479	0.1616	57
84	0.1507	0.1503	0.1616	57	0.1941	0.0272	0.1616	57
85	0.0815	0.8434	0.1616	57	0.0995	0.6251	0.1616	57
86	0.0730	0.9220	0.1616	57	0.0954	0.6769	0.1616	57
87	0.0662	0.9643	0.1616	57	0.0813	0.8452	0.1616	57
92	0.2577	0.0010	0.1616	57	0.3122	0.0000	0.1616	57
93	0.1098	0.4980	0.1616	57	0.1447	0.1837	0.1616	57
94	0.1335	0.2711	0.1630	56	0.1512	0.1546	0.1630	56
95	0.1253	0.3325	0.1616	57	0.2124	0.0117	0.1616	57
96	0.1842	0.0447	0.1630	56	0.2132	0.0123	0.1630	56
97	0.1320	0.2738	0.1616	57	0.1695	0.0755	0.1616	57
98	0.1044	0.5634	0.1616	57	0.1425	0.1970	0.1616	57
99	0.0695	0.9458	0.1616	57	0.0761	0.8960	0.1616	57
100	0.2650	0.0009	0.1645	55	0.2857	0.0003	0.1645	55
102	0.2395	0.0041	0.1660	54	0.2651	0.0010	0.1660	54
103	0.2289	0.0051	0.1616	57	0.2682	0.0005	0.1616	57
104	0.2522	0.0014	0.1616	57	0.2998	0.0001	0.1616	57

that for 11 of the 20 days the data were significantly different from log-normally distributed. Overall the data appear to be normally distributed.

During Experiment 3 midday temperature depression was measured on 15 days for Run 1 and 10 days for Run 2. On only 4 of the 25 days were data significantly different from normal (Table 4-5). On 7 out of 25 days the K-S tests showed that the data were significantly different from log-normally distributed. These data seem to be normally distributed.

Table 9-5.

Kolmogorov-Smirnov tests for midday temperature depression ($T_{o,max}$ - $T_{d,max}$), Experiment 3.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

	Tests for Normality.				Tests for Log-Normality.			
Name	KS d	Prob. D>d	c for P(0.10, N)	N	KS d	Prob. D>d	c for P(0.10, N)	N
Run 1.								
304	0.1718	0.0735	0.1630	56	0.1095	0.5129	0.1630	56
305	0.1870	0.0398	0.1630	56	0.2155	0.0121	0.1645	55
308	0.1379	0.2373	0.1630	56	0.1642	0.0977	0.1630	56
309	0.0720	0.9337	0.1630	56	0.0834	0.8304	0.1630	56
311	0.1209	0.3857	0.1630	56	0.1375	0.2403	0.1630	56
312	0.0710	0.9403	0.1630	56	0.0967	0.6721	0.1630	56
313	0.1010	0.6174	0.1630	56	0.0911	0.7420	0.1630	56
314	0.1160	0.4380	0.1630	56	0.0779	0.8859	0.1630	56
315	0.0624	0.9813	0.1630	56	0.0975	0.6618	0.1630	56
316	0.0666	0.9648	0.1630	56	0.1225	0.3701	0.1630	56
317	0.1058	0.5572	0.1630	56	0.4365	0.0000	0.1883	42
318	0.0790	0.8760	0.1630	56	0.1280	0.3175	0.1630	56
319	0.0757	0.9053	0.1630	56	0.1208	0.3877	0.1630	56
320	0.0720	0.9334	0.1630	56	0.0978	0.6574	0.1630	56
321	0.0806	0.8602	0.1630	56	0.1111	0.4940	0.1630	56
Run 2.								
329	0.1101	0.4947	0.1616	57	0.1499	0.1544	0.1616	57
330	0.2561	0.0011	0.1616	57	0.2811	0.0002	0.1616	57
331	0.1186	0.3990	0.1616	57	0.1891	0.0339	0.1616	57
332	0.1068	0.5339	0.1616	57	0.0925	0.7138	0.1616	57
333	0.1472	0.1693	0.1616	57	0.1085	0.5130	0.1616	57
334	0.1695	0.0755	0.1616	57	0.1341	0.2568	0.1616	57
335	0.0941	0.6935	0.1616	57	0.1356	0.2457	0.1616	57
336	0.0667	0.9615	0.1616	57	0.0760	0.8973	0.1616	57
337	0.1579	0.1167	0.1616	57	0.2168	0.0094	0.1616	57
338	0.1078	0.5222	0.1616	57	0.1877	0.0360	0.1616	57

Data for evaporation at the 57 locations were also taken for 25 days during Experiment 3. For 18 days the data were significantly different from normal in distribution (Table 9-6). For 16 days the data were not log-normally distributed at the 10% level of significance. These data appear to follow neither a normal nor log-normal distribution.

Table 9-6.

Kolmogorov-Smirnov tests for evaporation (mm), Experiment 3.

Small values of probability indicate that the cumulative distribution of the data is significantly different from the normal distibution. For tests of log-normality, sample values that were zero or negative were excluded from analysis.

	Tests for Normality.				Tests for Log-Normality.			
Name	KS d	Prob. D>d	c for P(0.10, N)	N	KS d	Prob. D>d	c for P(0.10, N)	N
Run 1.								
304	0.6686	0.0000	0.1645	55	0.6043	0.0000	0.1645	55
305	0.2180	0.0097	0.1630	56	0.1483	0.1705	0.1630	56
308	0.1876	0.0388	0.1630	56	0.1935	0.0302	0.1630	56
309	0.1014	0.6121	0.1630	56	0.1085	0.5251	0.1630	56
311	0.1590	0.1178	0.1630	56	0.1548	0.1368	0.1630	56
312	0.1620	0.1057	0.1630	56	0.1092	0.5161	0.1630	56
313	0.2090	0.0150	0.1630	56	0.1314	0.2881	0.1630	56
314	0.1710	0.0757	0.1630	56	0.1296	0.3040	0.1630	56
315	0.2028	0.0200	0.1630	56	0.1823	0.0484	0.1630	56
316	0.1734	0.0689	0.1630	56	0.1880	0.0383	0.1630	56
317	0.1152	0.4472	0.1630	56	0.1702	0.0781	0.1630	56
318	0.2196	0.0090	0.1630	56	0.2911	0.0002	0.1630	56
319	0.2705	0.0006	0.1630	56	0.3072	0.0001	0.1630	56
320	0.2854	0.0002	0.1630	56	0.2885	0.0002	0.1630	56
321	0.3046	0.0001	0.1630	56	0.3141	0.0000	0.1630	56
Run 2.								
329	0.6273	0.0000	0.1676	53	0.5372	0.0000	0.1676	53
330	0.1766	0.0570	0.1616	57	0.2553	0.0012	0.1616	57
331	0.0974	0.6514	0.1616	57	0.1435	0.1913	0.1616	57
332	0.1508	0.1497	0.1616	57	0.1124	0.4671	0.1616	57
333	0.1949	0.0263	0.1616	57	0.1131	0.4599	0.1616	57
334	0.2297	0.0067	0.1660	54	0.1918	0.0377	0.1660	54
335	0.2447	0.0028	0.1645	55	0.1780	0.0613	0.1645	55
336	0.2225	0.0086	0.1645	55	0.1976	0.0273	0.1645	55
337	0.2019	0.0192	0.1616	57	0.2354	0.0036	0.1616	57
338	0.1824	0.0450	0.1616	57	0.2397	0.0029	0.1616	57

Variograms.

Sample variograms were calculated for each variable using different class widths. Class widths reported below are those that provided the most detail (smallest class width) without causing the variogram values to become excessively noisy. After trying classes with both equal class widths (unequal numbers of pairs in each class) and equal numbers of pairs in each class (unequal class widths), it was decided to use classes with equal class widths since no discernible improvement resulted from using classes with equal numbers of pairs.

Because the field was relatively long (220 m) and narrow (52 m) it was difficult to estimate the degree of anisotropy in the data. Therefore the data were all assumed to be isotropic (variograms have the same value for a given separation distance, h, regardless of the direction of h).

Catch Can Depths. Variograms calculated for catch can depths showed about the same range of autocorrelation (20 m) for each irrigation in Experiment 2 (Figure 9-1, top). The spherical model was fit by eye with a nugget of zero and range of 20 m. The spherical model is:

$$\gamma(h) = \begin{cases} C_0 + C_1[1.5h/a - 0.5(h/a)^{1/3}], & 0 \leq h \leq a \\ C_0 + C_1, & h > a \end{cases} \qquad [9\text{-}9]$$

where C_0 is the nugget, C_1 is the sill minus the nugget, a is the range, and h is the separation distance. The sill value was different for variograms from each irrigation. In contrast, the relative variograms for the 3 irrigations were quite similar, at least up to 30 m separation distance (Figure 9-1, bottom). The range and nugget were again chosen as 20 m and zero, respectively but a common sill value of 0.057 could be chosen for all 3 irrigations. Between 30 and 45 m the variogram values for irrigations 1 and 3 were very similar but those for irrigation 2 were higher. For kriging the most useful variogram model information is contained within the range, so the discrepancies between variogram values at separation distances above 20 m should be of little concern. The class width was 7.5 m and there were 7 classes for these variogram calculations.

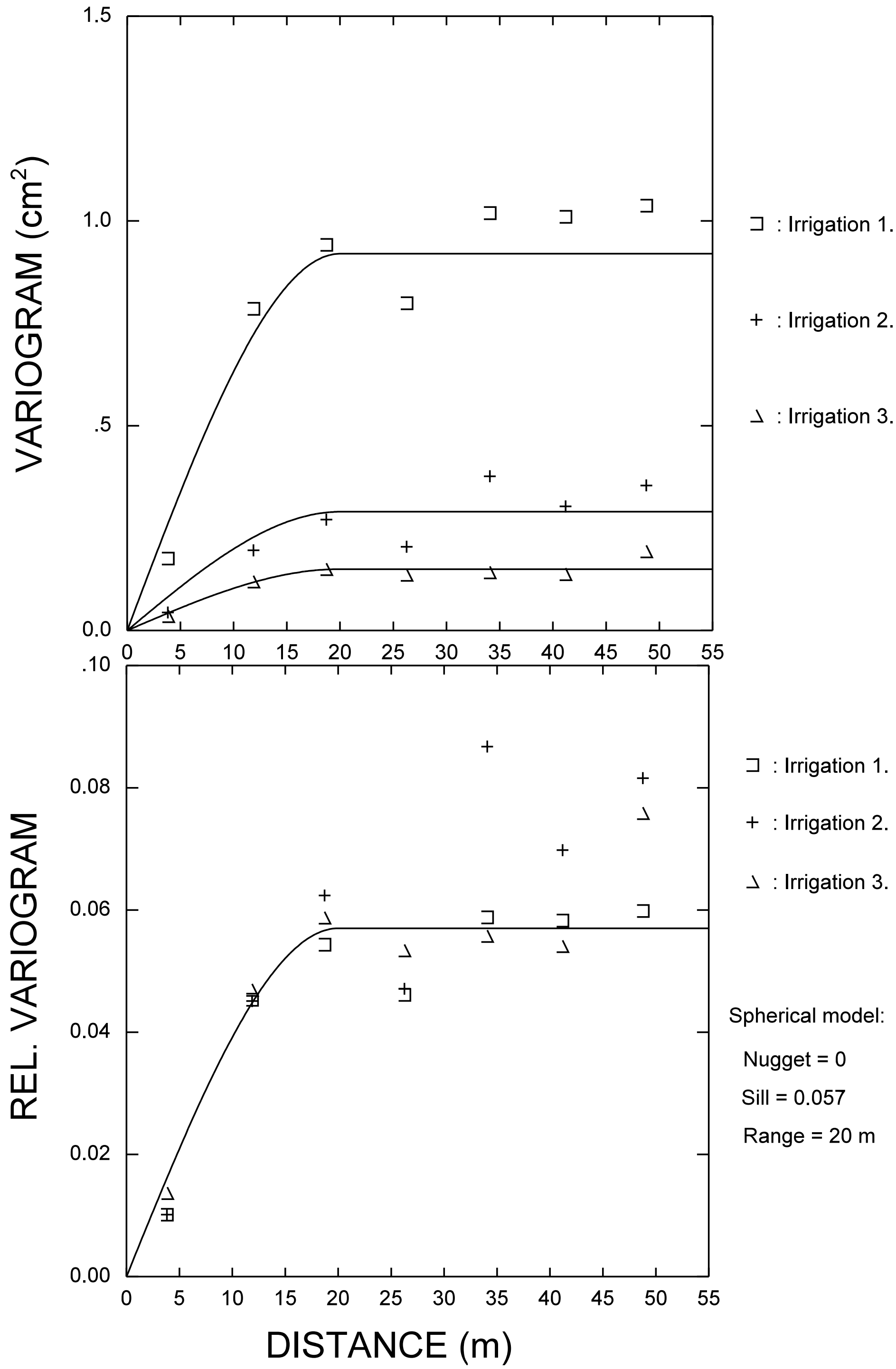

Figure 9-1. (top) Variograms for catch can depths from 3 irrigations, Experiment 2. (bottom) Relative variograms for the same 3 irrigations.

<u>Change in Storage</u>. Variograms calculated for the change in storage due to irrigation showed no discernible spatial dependence for any of the 3 irrigations in Experiment 2 (Figure 9-2). Change in storage data were essentially random and there is no point in fitting a variogram model. Using smaller class widths, e.g. 5 m, for the variogram calculations was not successful in revealing spatial dependence at small separation distances.

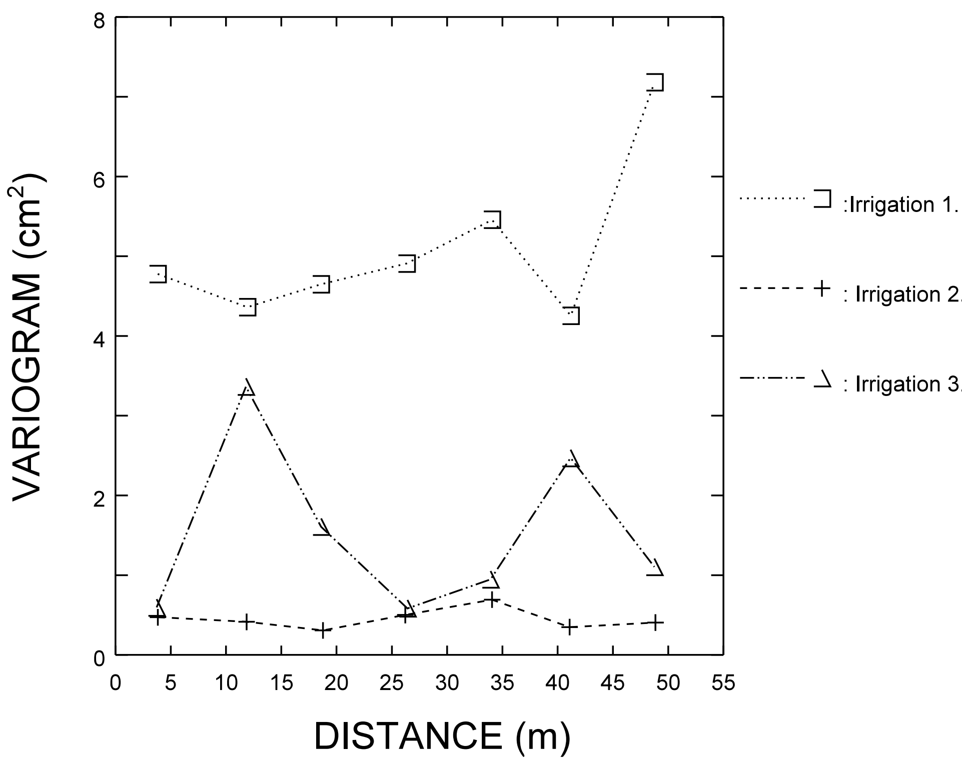

Figure 9-2. Sample variograms for the change in storage due to irrigation, 3 irrigations, Experiment 2.

Profile Water Content. Relative variograms were calculated for profile water contents measured on 16 days after irrigation during Experiment 2. Data from the 5 locations within 1.5 m of the field edge were omitted from the analysis to avoid edge effects. A class width of 5 m gave the most informative variogram values. For separation distances below 35 m a linear model, with a nugget of 0.014 (the relative variogram is unitless) and slope of 0.00102, appeared appropriate (Figure 9-3). There is a change in depth to sand (increasing from the SE corner to the NW corner) which may explain the odd behavior of the variogram at separation distances above 35 m. Since the linear model is only appropriate up to 35 m it would be important to use a kriging neighborhood of 35 m or less. Considering the large number of separate days data that were combined to create the relative variogram, it is remarkable how little dispersion occurred about the mean sample relative variogram values. This is a result of the high degree of time invariance for these data.

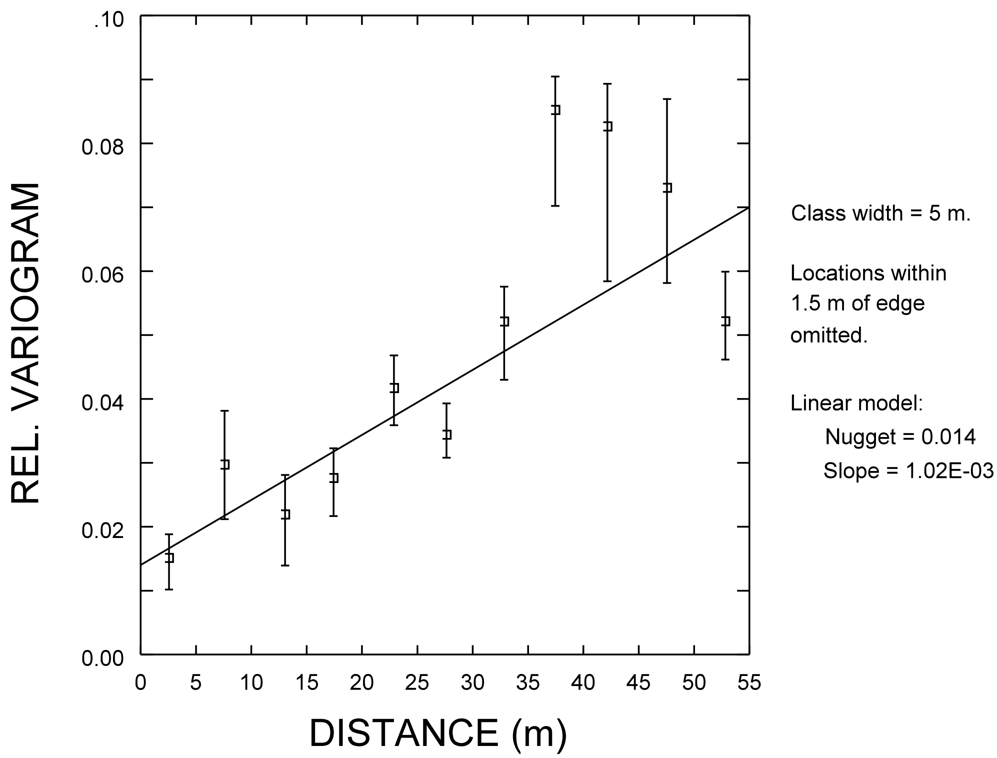

Figure 9-3. Sample relative variogram values for profile water content after irrigation, Experiment 2. Squares represent the average value for all days, error bars show maximum and minimum values.

Midday Temperature Depression. The maximum temperature difference between the reference dry soil and drying soil, ($T_{o,max}$ - $T_{d,max}$), was measured daily for Experiments 2 and 3. For Experiment 2 the drying soil temperature was obtained by scanning the area around the access tubes in a circular pattern while making 50 readings with the infrared thermometer, then taking the average reading. For Experiment 3 drying soil temperatures were taken by pointing the infrared thermometer at the top of each ML and averaging 10 readings made without moving the point of focus of the thermometer. Despite the differences in technique there were more similarities than differences between the 2 experiments in the spatial variability of ($T_{o,max}$ - $T_{d,max}$).

Sample variograms for Irrigations 1 and 2 of Experiment 2 are shown in Figure 9-4. For the first 2 to 5 days after irrigation there appears to be some spatial dependence especially for the Irrigation 2 data. The range is difficult to discern but is perhaps 10 m. On later days there appears to be little spatial dependence. The instability of spatial dependence is illustrated by the relative variogram plot of Figure 9-5. It was not possible to reduce the sample variograms for different days to a single relative variogram model. The average relative variogram showed a small degree of spatial dependence. For these analyses and those to follow, the locations within 1.5 m of the field edge were omitted to reduce edge effects. Except for days immediately after irrigation these data could be considered spatially independent.

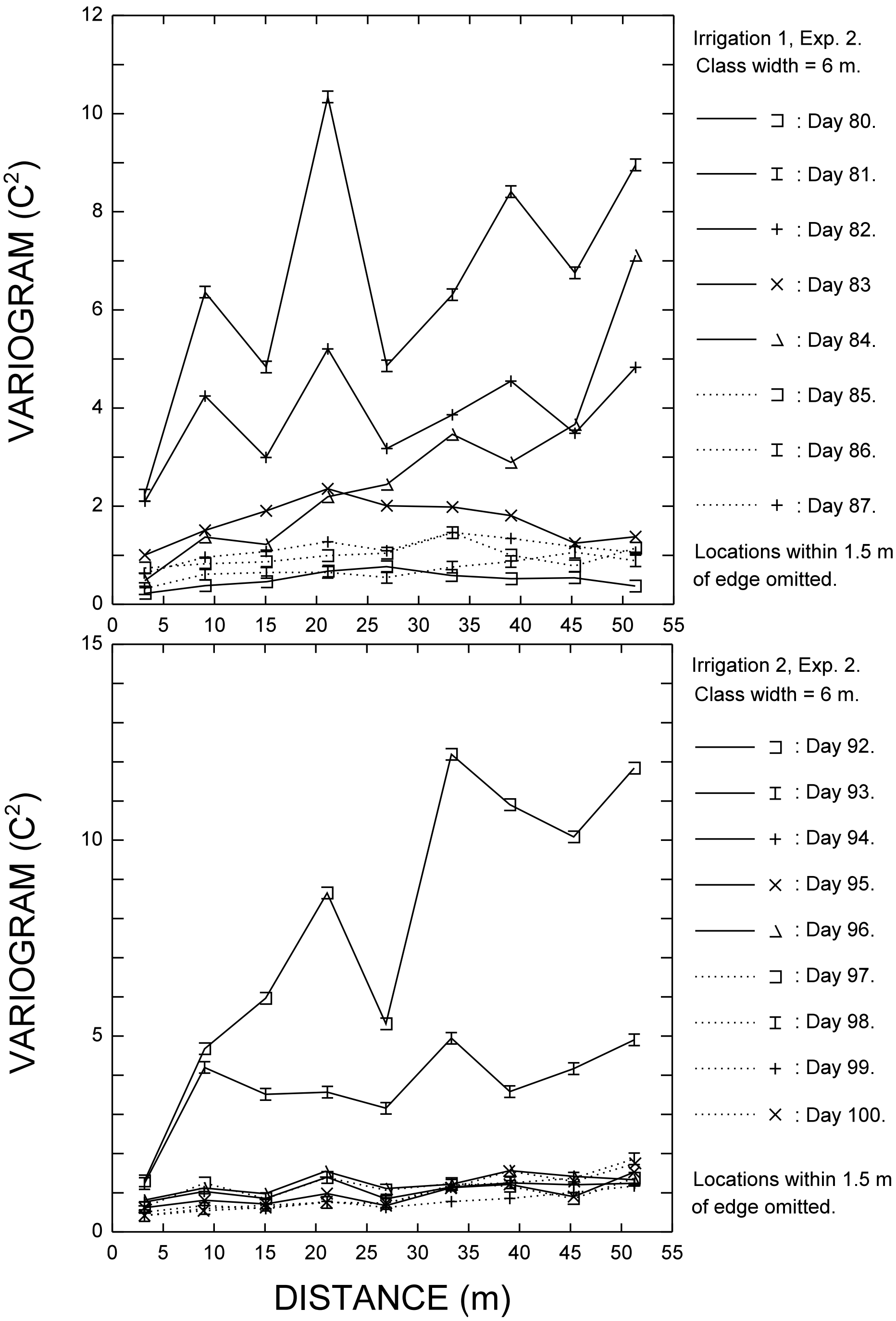

Figure 9-4. Sample variograms for ($T_{o,max}$ - $T_{d,max}$), (top) after Irrigation 1, (bottom) after Irrigation 2, Experiment 2.

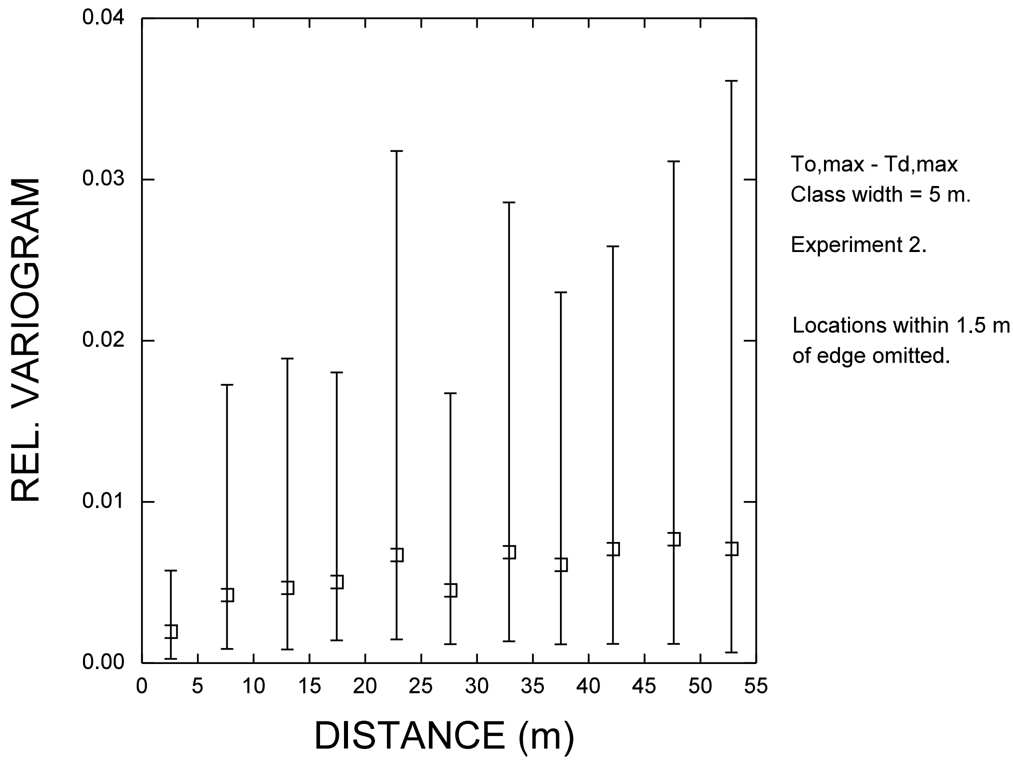

Figure 9-5. Relative variograms for ($T_{o,max}$ - $T_{d,max}$) after Irrigations 1 and 2, Experiment 2.

Experiment 3 data showed even less spatial dependence than did ($T_{o,max}$ - $T_{d,max}$) data from Experiment 2. As for Experiment 2, the variance tended to be higher for data from days immediately after irrigation, declining thereafter (Figure 9-6). Again, it was not possible to find a single variogram model that would fit relative variograms from all days (Figure 9-7). The average relative variogram also showed no spatial dependence.

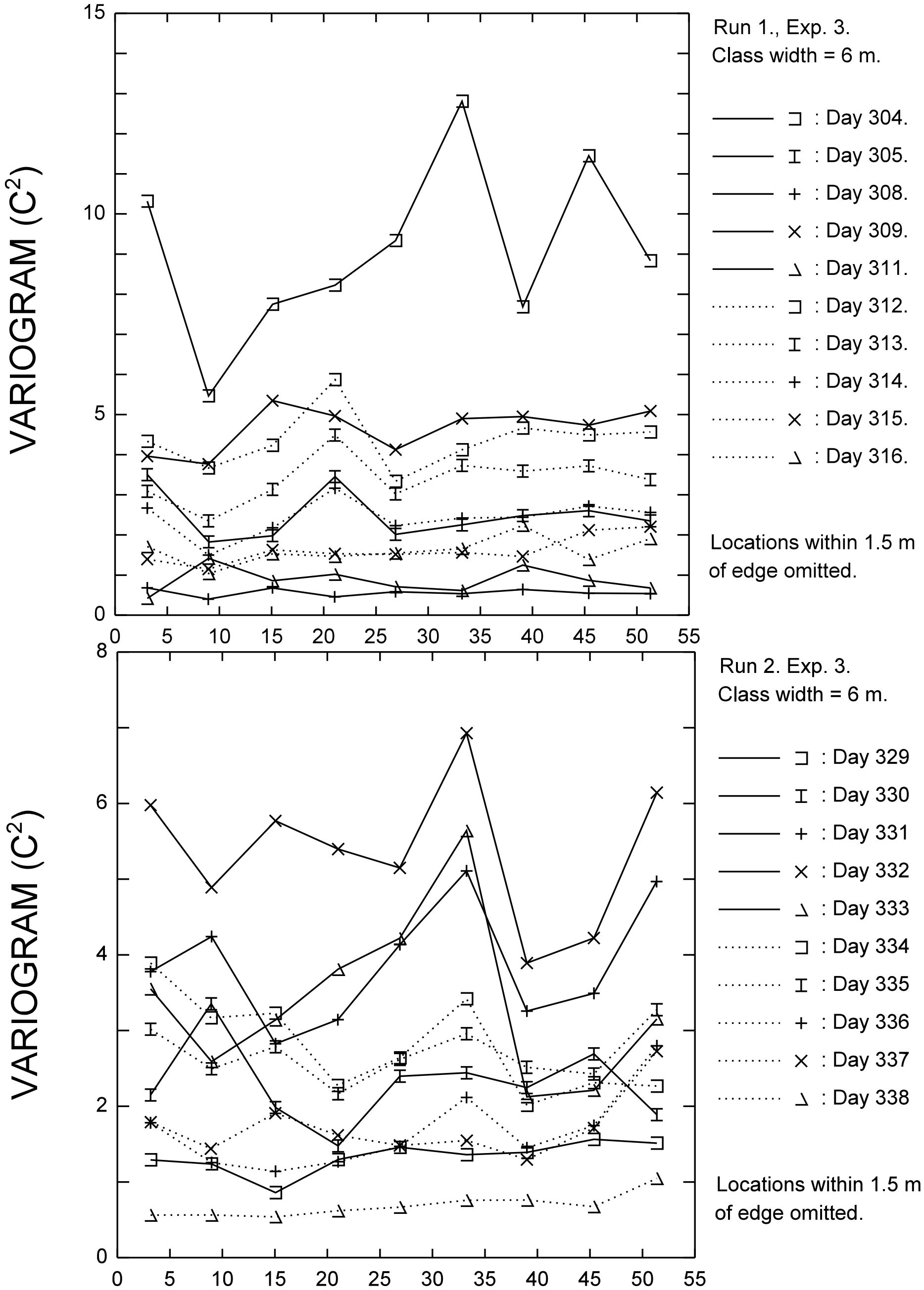

Figure 9-6. Sample variograms for ($T_{o,max}$ - $T_{d,max}$) from Experiment 3, (top) Run 1 and (bottom) Run 2.

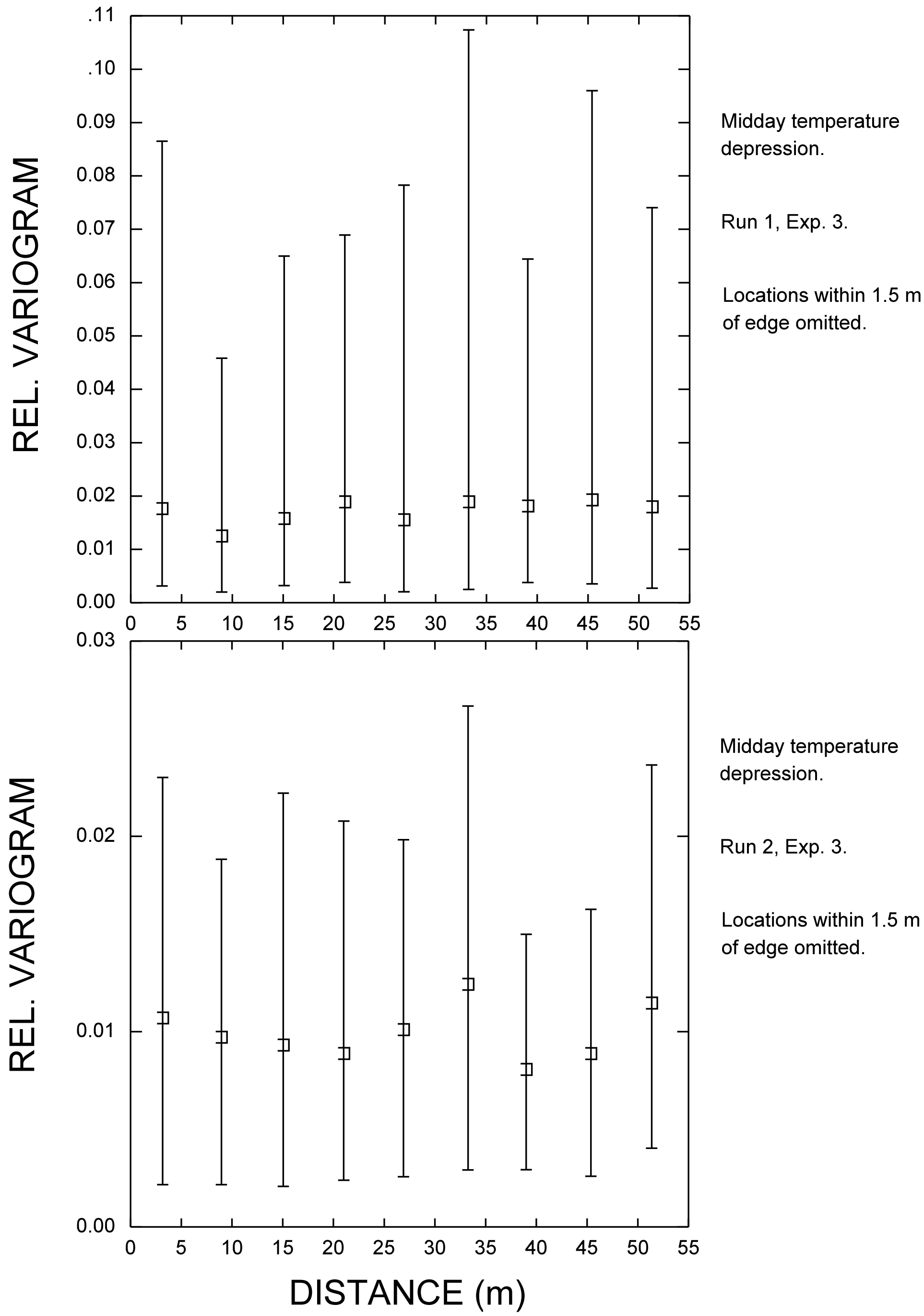

Figure 9-7. Sample relative variograms for ($T_{o,max}$ - $T_{d,max}$) for Experiment 3, (top) Run 1 and (bottom) Run 2.

Evaporation. Daily evaporation was measured with ML's at the 57 field locations during Runs 1 and 2 of Experiment 3. As for temperature data, sample variograms based on evaporation data showed little spatial dependence (Figure 9-8). On the 4th and 5th days after the second irrigation there appears to be some spatial dependence but this was considered to be an anomaly since on all other days no spatial dependence was observed.

For both Run 1 and Run 2 sample relative variograms showed the same lack of spatial dependence (Figure 9-9). These evaporation data appear to be randomly distributed with no spatial dependence. Because both temperature and evaporation data are spatially independent there is no reason to attempt kriging or cokriging with these data.

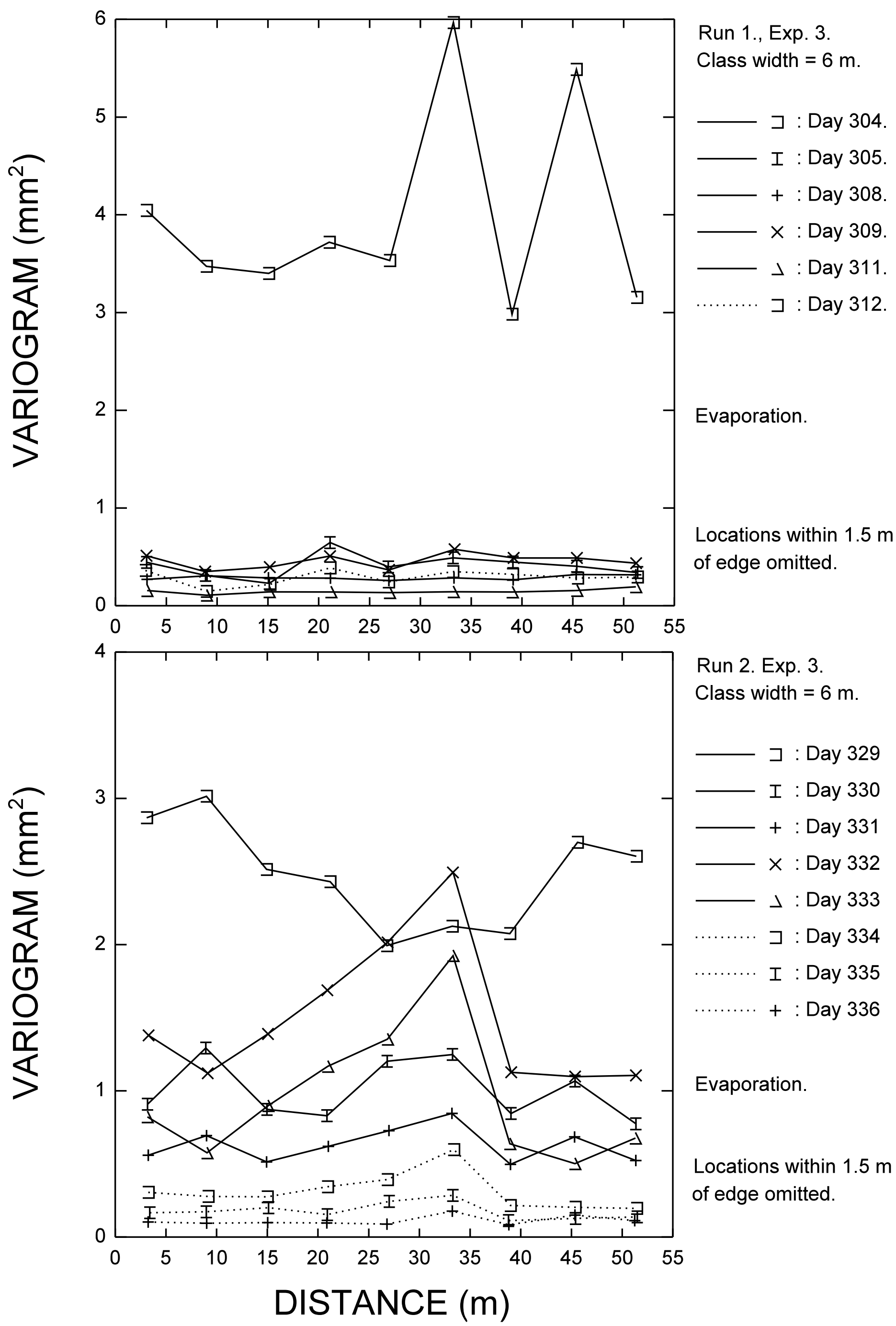

Figure 9-8. Sample variograms for evaporation, (top) Run 1 and (bottom) Run 2, Experiment 3.

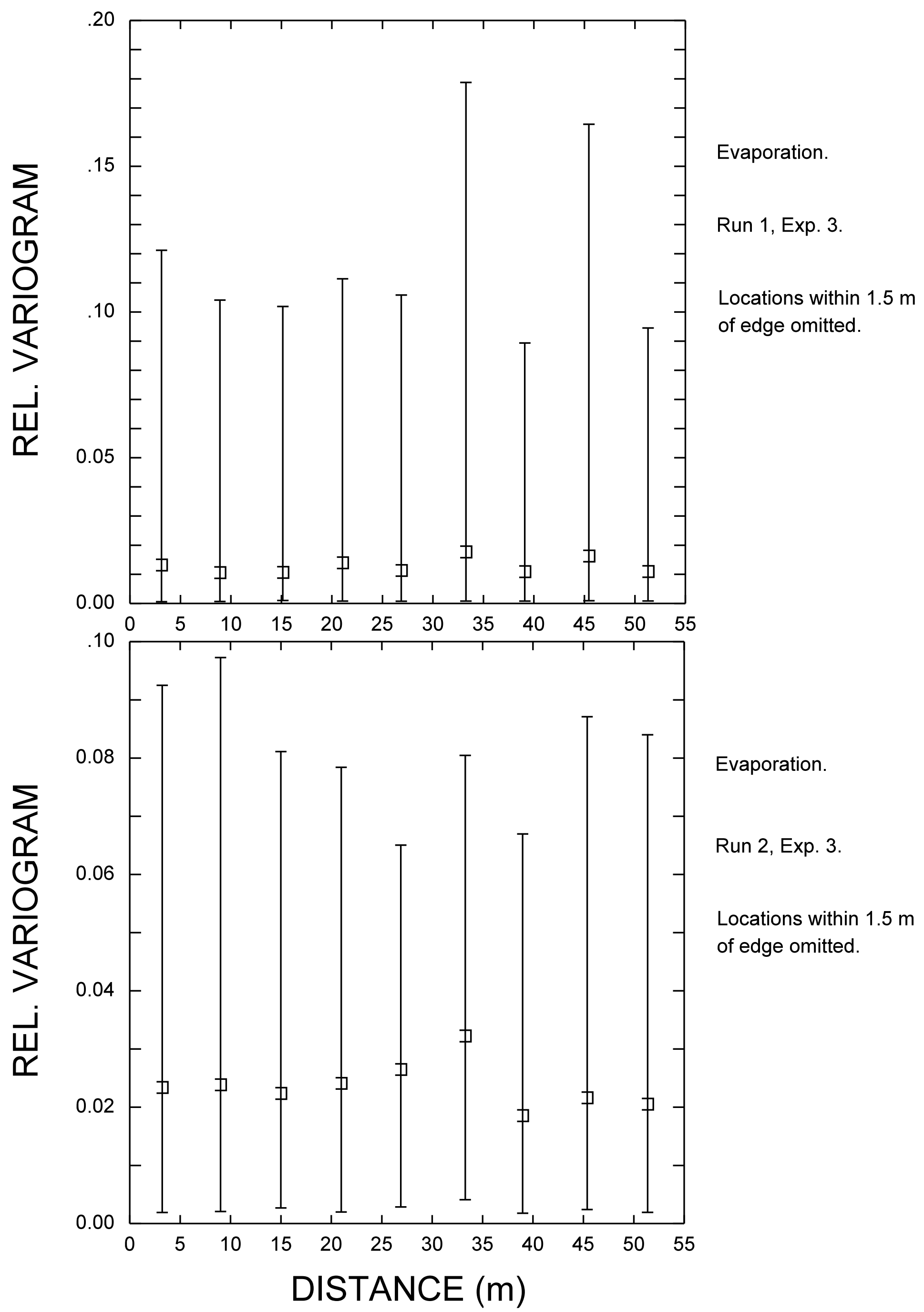

Figure 9-9. Sample relative variograms for evaporation, Experiment 3, (top) Run 1 and (bottom) Run 2.

Kriging.

Ordinary (punctual) kriging was done using the GEOEAS program KRIGE. Before kriging, the variogram models were examined by cross-validation using the GEOEAS program XVALID. In the usual kriging procedure the estimated value at the sample location is, by definition, equal to the sample value. Cross-validation is a procedure in which kriging is used to estimate the sample value at each sample location as if the value were not already known. The estimated and sample values are then compared. If the variogram model is a good one then the average estimated value should be close to the average sample value. The error, $[Z^*(x_i) - Z(x_i)]$, should be close to zero.

In practice a dimensionless error, called the reduced error, ϵ_r, is defined as:

$$\epsilon_r = [Z^*(x_i) - Z(x_i)]/\sigma(x_i) \qquad [9\text{–}10]$$

where $\sigma(x_i)$ is the kriging standard deviation. The reduced error should have a mean close to zero and should have variance close to one if the variogram model is good (Viera et al. 1983). In the GEOEAS nomenclature the reduced error is called the zscore and its mean and standard deviation are reported.

Catch Can Depths. The usefulness of the relative variogram depicted in Figure 9-1 (bottom) was tested as follows. Recall that in Chapter 8 the ranked sample locations were plotted vs. average relative difference. Examination of Figure 8-1 shows that location 17 was close to the mean relative difference and was stable over time. The sample values at location 17 were 3.99, 2.03 and 1.64 cm for irrigations 1, 2 and 3, respectively. Assuming a zero nugget and constant range of 20 m, the spherical relative variogram model was scaled for each irrigation by multiplying the sill value by the square of the mean value of catch can depth for that irrigation, with the mean value set equal to the value at location 17. Thus this procedure was also a test of the time invariance concept and location 17 as representative of the mean.

Cross-validation resulted in mean reduced errors of 0.043, 0.075 and 0.048 for the 3 irrigations (Table 9-7). The mean sample values were accurately reproduced with the largest error amounting to 1.6% of the mean. The standard deviation of the reduced errors was close to 1 with the largest difference being 0.19 for Irrigation 3. Plotting of estimated vs. sample values for the 3 irrigations shows that points are clustered about the 1 to 1 line, a result of the mean being successfully reproduced (Figure 9-10).

Table 9-7.

Cross-validation for catch can depth data, 3 irrigations, Experiment 2. Ordinary kriging using GEOEAS.

Search Ellipse Parameters:		Distance type :	Euclidean
R Major :	35.0	Num. sectors :	1
R Minor :	35.0	Max pts/sector:	8
Angle :	0.0	Min pts to use:	1
Min Dist:	0.0	Empty sectors :	0

Variogram Model: Spherical

Irrigation	Nugget	Sill	Range (m)
1	0.0	0.907	20.0
2	0.0	0.236	20.0
3	0.0	0.153	20.0

Irrigation 1.

	Variable	Estimate	Difference	Kriging Std. Dev.	Zscore
Minimum	2.620	2.810	-1.238	.261	-1.559
25th %tile	4.035	4.122	-.214	.361	-.474
Median	4.347	4.362	-.065	.487	-.119
75th %tile	4.825	4.675	.323	.725	.509
Maximum	5.615	5.103	2.292	1.134	2.021
N	52	52	52	52	52
Mean	4.351	4.393	.042	.566	.043
Std. Dev.	.694	.477	.592	.237	.815

Irrigation 2.

	Variable	Estimate	Difference	Kriging Std. Dev.	Zscore
Minimum	.805	.999	-.768	.133	-1.896
25th %tile	2.082	2.214	-.166	.184	-.720
Median	2.254	2.254	.041	.249	.171
75th %tile	2.375	2.335	.172	.370	.596
Maximum	2.831	2.718	1.480	.579	2.558
N	52	52	52	52	52
Mean	2.192	2.229	.036	.289	.075
Std. Dev.	.404	.280	.354	.121	1.005

Irrigation 3.

	Variable	Estimate	Difference	Kriging Std. Dev.	Zscore
Minimum	.819	.907	-.607	.107	-2.308
25th %tile	1.543	1.597	-.177	.148	-.908
Median	1.664	1.701	.020	.200	.129
75th %tile	1.850	1.784	.169	.298	.667
Maximum	2.208	2.165	.734	.466	2.782
N	52	52	52	52	52
Mean	1.669	1.683	.015	.233	.048
Std. Dev.	.292	.205	.276	.097	1.192

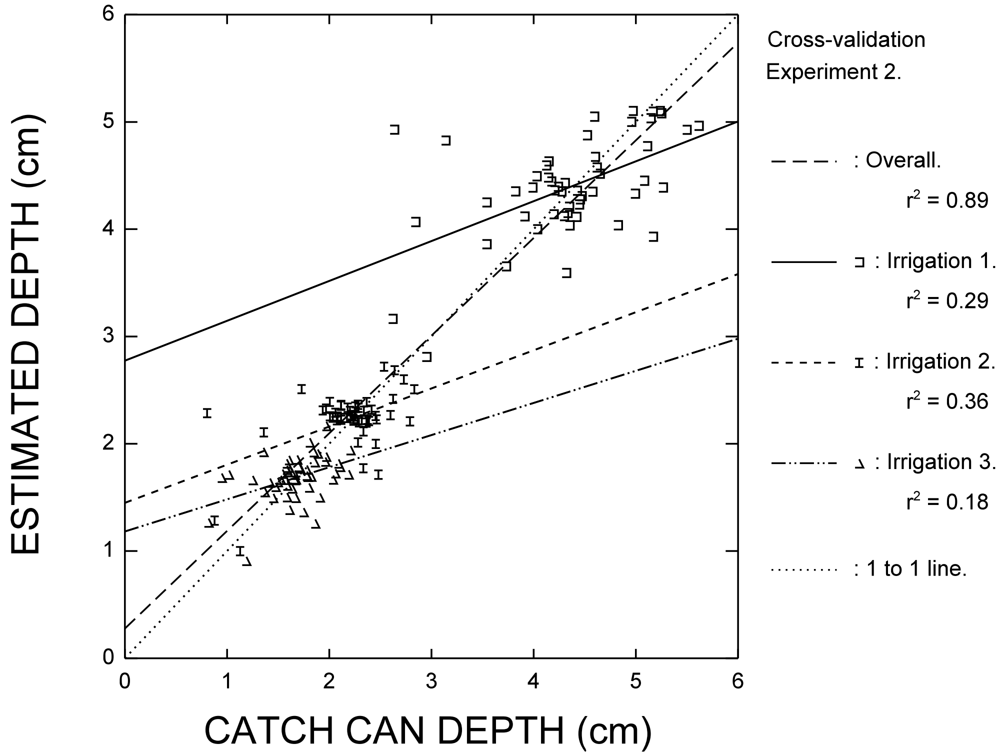

Figure 9-10. Kriging estimates from cross-validation vs. actual catch can depths for 3 irrigations, Experiment 2.

A disturbing tendency is that extreme values were estimated as being closer to the mean than was actually the case. This results in the regression lines having a low slope, quite different from the 1 to 1 line. This is partly a result of the fact that kriging is a smoothing process and the kriging variance will always be smaller than the sample variance (Warrick et al. 1986). It may also be a result of undersampling in some areas of the field. The r^2 values seem low but this is not unusual. For cross-validation on 11 different variables Viera et al. (1983) found r^2 values ranging from 0.30 to 0.48 for the regression of estimated vs. sample values. The overall r^2 value of 0.89 and regression line close to the 1 to 1 line were a result of the fact that the mean depth varied widely from irrigation to irrigation and that the mean was well estimated. On the basis of the values for mean and standard deviation of the reduced error, the procedure described above, for using the relative variogram and a time invariant sample location representing the mean, appears to be a good one.

Kriging was done on a 5 by 5 m grid for the Irrigation 1 catch can depths using a 35 m circular search pattern. The contour plot of kriged estimates is realistic (Figure 9-11) and, as expected, slightly smoother than the contour plot of measured values (Figure 9-13). The contour plot of measured values was created using an inverse weighting method according to the square of the distance to the 10 nearest neighbors. The kriging standard deviations range from near zero in areas of the field that were heavily sampled to 1.3 cm at the NE corner which was far from any samples (Figure 9-12).

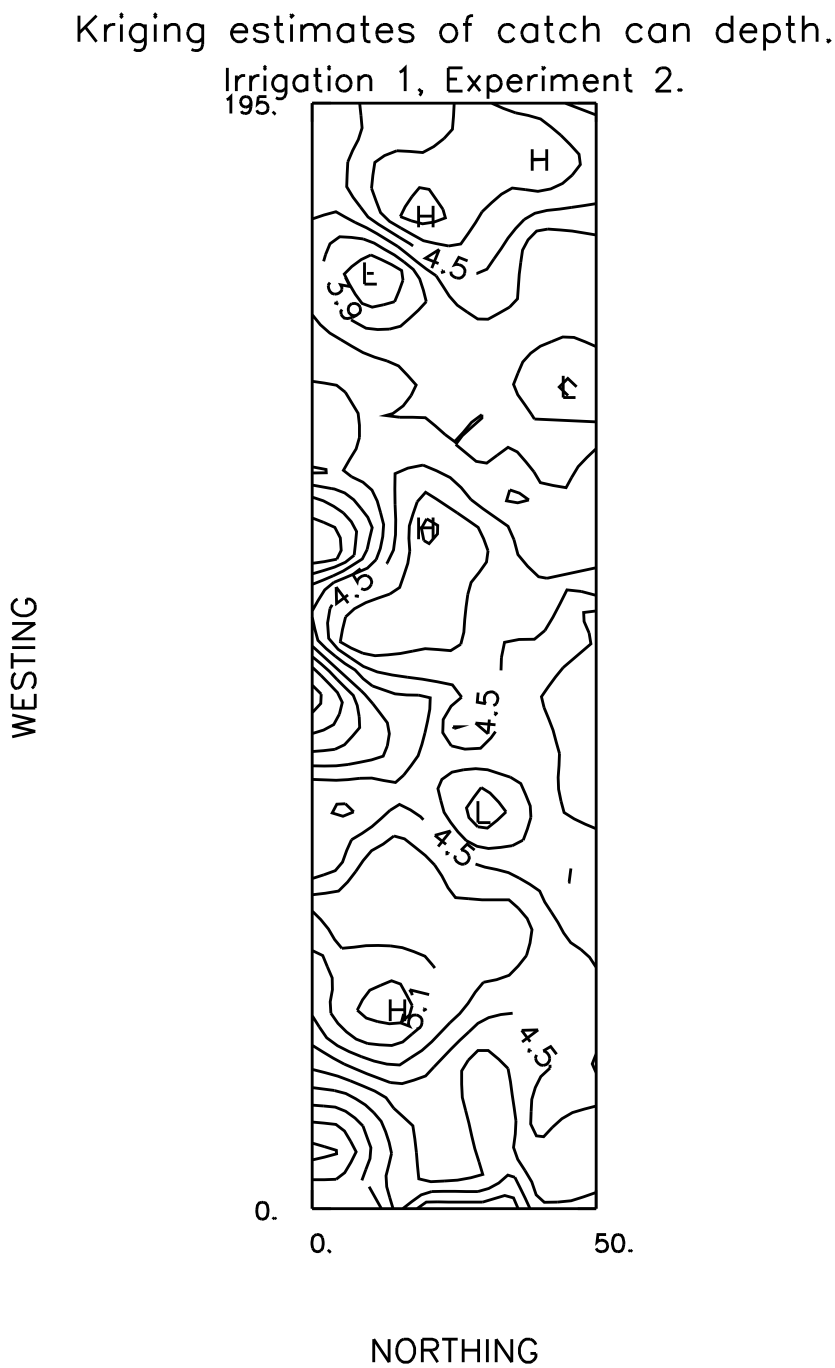

Figure 9-11. Contours of kriging estimates of catch can depths, Irrigation 1, Experiment 2. Contour lines are on 0.3 cm intervals.

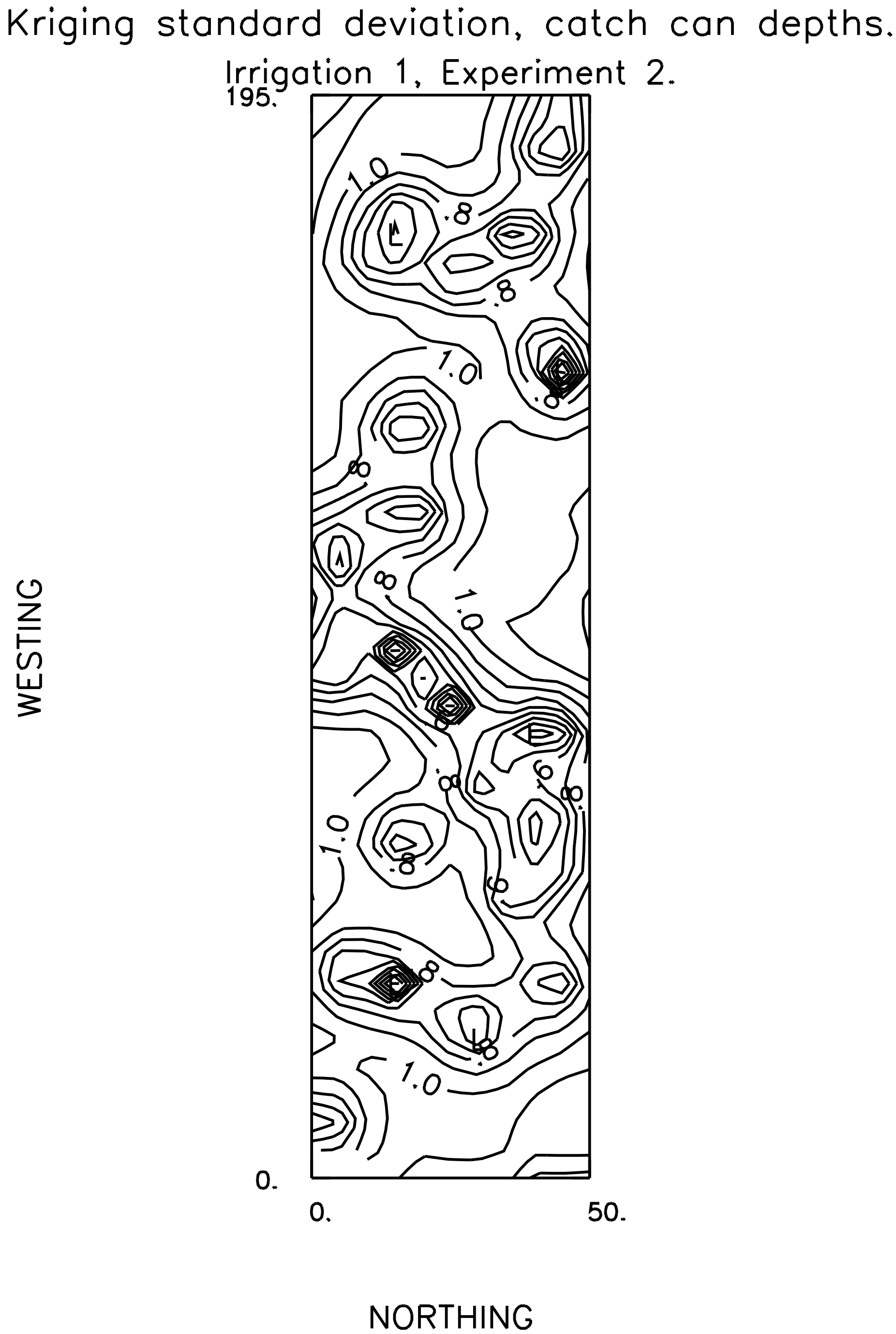

Figure 9-12. Contours of kriging standard deviation, catch can depths, Irrigation 1, Experiment 2. Contours are on 0.1 cm intervals.

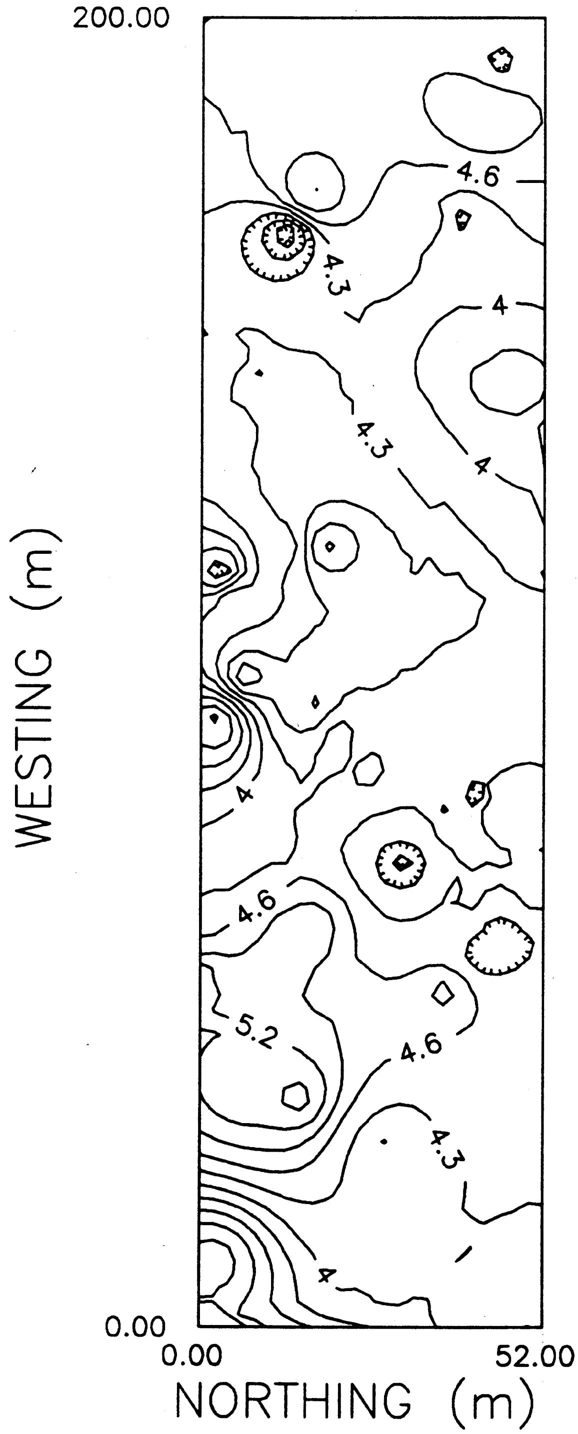

Figure 9-13. Contours of measured catch can depths, Irrigation 1, Experiment 2. Contours are on 0.3 cm intervals.

Profile Water Content. The relative variogram model depicted in Figure 9-3 was tested by cross-validation using data from 5 days. Examination of Figure 8-4 revealed that location 21 was closest to the mean profile water content and was stable over time. The value of profile water content at location 21 was therefore used to represent the mean profile water content. The relative variogram model was scaled to fit each day's data by multiplying the nugget and sill values of the relative linear model by the square of the mean (Table 9-8).

In GEOEAS nomenclature the sill of a linear model is defined as the slope multiplied by the range. Therefore the sill values for GEOEAS in Table 9-8 are the result of multiplying the slope by an appropriate range (35 m was used) and then multiplying by the square of the mean. Since the linear model does not have a maximum value (as do the spherical and gaussian models for example) it is inappropriate to apply the sill concept here. The sill and range are only used in GEOEAS to define the slope of the linear model.

The reduced mean error and standard deviation from cross-validation, using the models in Table 9-8, were in all cases close to zero and 1, respectively (Table 9-9). The largest difference between mean estimated and measured profile water content was 0.5%. Histogram plots of the frequency of the error term showed the error term to be approximately normally

Table 9-8.

Linear variogram model parameters for profile water content scaled from relative variogram nugget and slope using depth at location 21 as representative of the mean depth.

Day	Mean Depth	Relative Nugget	Relative Slope	For GEOEAS KRIGE & XVALID Nugget	Sill	Range (m)
77	22.70	0.014	0.00102	7.22	18.4	35
80	27.57	0.014	0.00102	10.64	27.1	35
81	26.87	0.014	0.00102	10.11	25.8	35
82	26.61	0.014	0.00102	9.91	25.3	35
83	26.16	0.014	0.00102	9.58	24.4	35
85	25.57	0.014	0.00102	9.15	23.3	35
90	26.13	0.014	0.00102	9.56	24.4	35
92	27.61	0.014	0.00102	10.67	27.2	35
93	27.06	0.014	0.00102	10.25	26.1	35
94	26.99	0.014	0.00102	10.20	26.0	35
95	26.98	0.014	0.00102	10.19	26.0	35
96	26.58	0.014	0.00102	9.89	25.2	35
98	26.57	0.014	0.00102	9.88	25.2	35
100	26.34	0.014	0.00102	9.71	24.8	35
102	26.90	0.014	0.00102	10.13	25.8	35
103	26.67	0.014	0.00102	9.96	25.4	35

distributed. Plotting the estimated vs. measured profile water contents showed the points to be clustered about the mean (Figure 9-14). Points for different days tended to plot in the same positions showing the stability of profile water content over time. This is a second case in which the use of a relative variogram model, in conjunction with a location known to be representative of the mean, has proven to be useful.

Table 9-9.

Cross-validation results for ordinary kriging on profile water contents for selected days, Experiment 2, using linear variogram models given in Table 9-8.

				Kriging	
Day 80.	Variable	Estimate	Difference	Std. Dev.	Zscore
N	52	52	52	52	52
Mean	27.946	28.009	.063	4.309	.001
Std. Dev.	5.586	3.542	5.008	.585	1.130
Day 82.					
N	52	52	52	52	52
Mean	27.454	27.524	.071	4.159	.003
Std. Dev.	5.542	3.610	4.762	.564	1.114
Day 85.					
N	52	52	52	52	52
Mean	27.173	27.282	.109	3.997	.009
Std. Dev.	5.394	3.605	4.570	.542	1.114
Day 92.					
N	52	52	52	52	52
Mean	27.821	27.944	.123	4.316	.011
Std. Dev.	5.464	3.692	4.437	.586	1.003
Day 100.					
N	51	51	51	51	51
Mean	26.892	27.015	.124	4.118	.013
Std. Dev.	5.241	3.540	4.246	.541	1.005
Day 102.					
N	50	50	50	50	50
Mean	27.439	27.567	.128	4.218	.013
Std. Dev.	5.295	3.535	4.318	.552	1.002

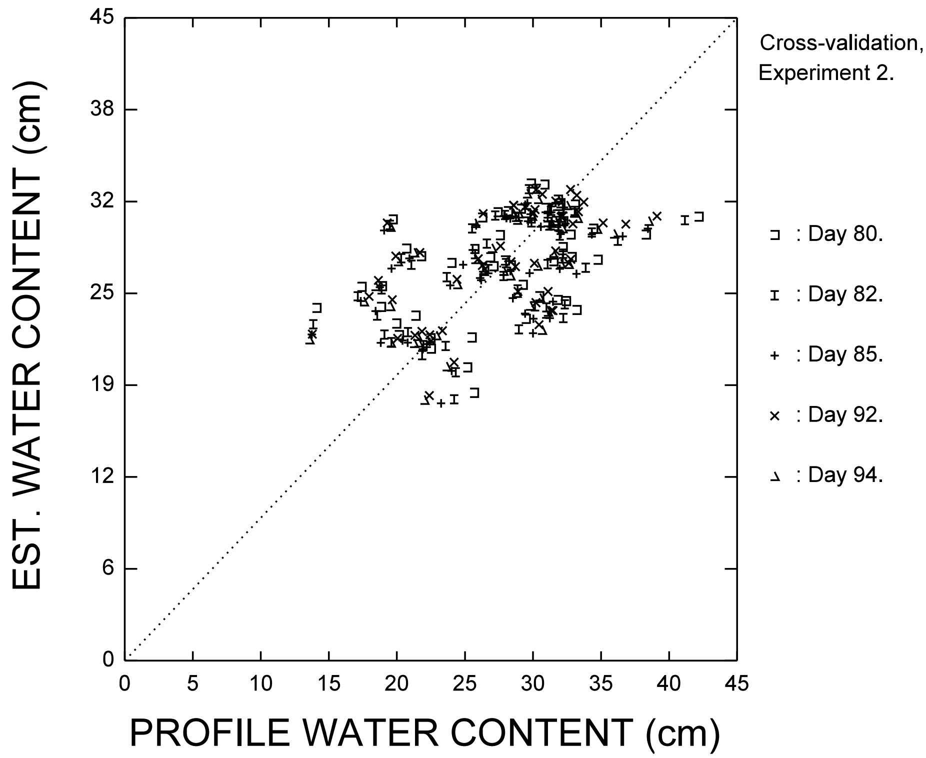

Figure 9-14. Kriging estimates from cross-validation vs. measured profile water contents for 5 days, Experiment 2.

Kriging was done for day 92 as an example. Kriging was done on a 5 by 5 m grid using a 35 m circular radius search pattern. The kriged estimates are realistic and reflect the the fact that depth to sand was greater on the west end of the field where estimates were up to 36 cm profile water content (Figure 9-15). By contrast, estimates at the east end of the field were as low as 20 cm. A contour map of measured values was produced using the same method as for catch can depths. Contours of measured values were quite similar to those of kriged values but less smooth (Figure 9-17). The kriging standard deviation was evenly distributed with a few areas approaching zero where sampling was heavy and with values of 7 and 8 cm in the SW and NE corners, respectively, where sampling was very light (Figure 9-16).

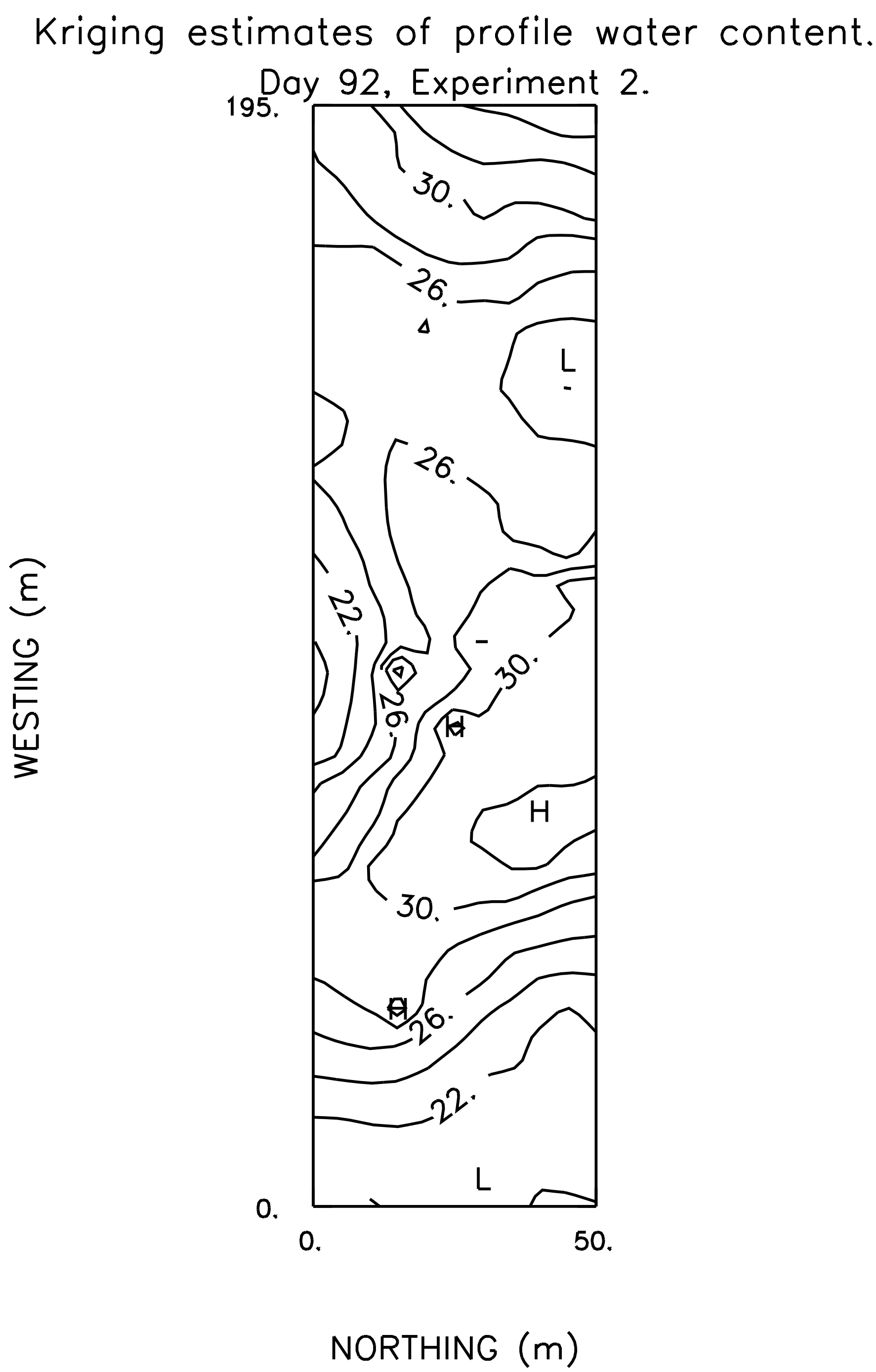

Figure 9-15. Kriging estimates of profile water content for day 92, Irrigation 2, Experiment 2. Contour intervals are on 2 cm increments.

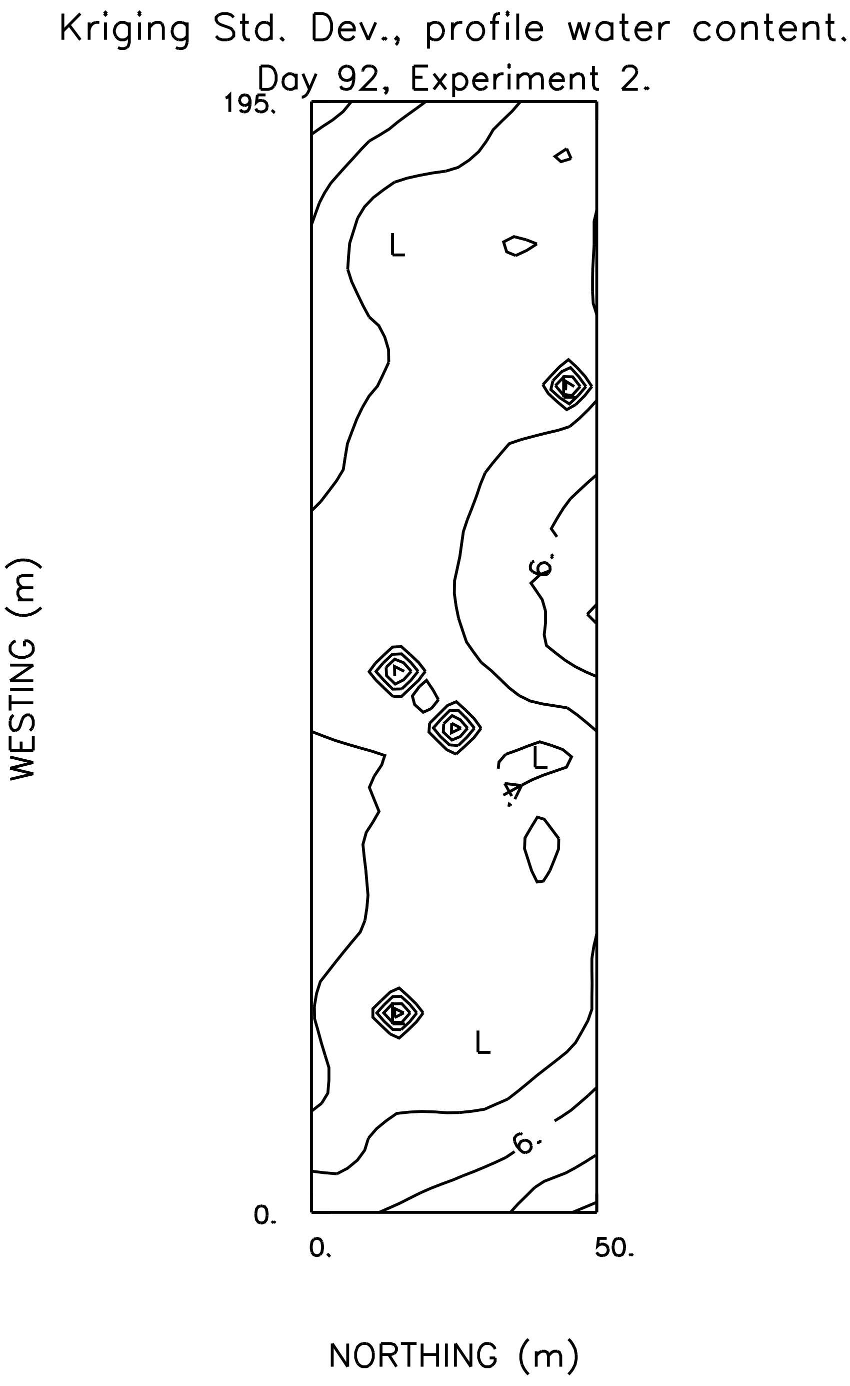

Figure 9-16. Kriging standard deviation for day 92 profile water content estimation, Irrigation 2, Experiment 2. Contour intervals are on a 1.0 cm increment.

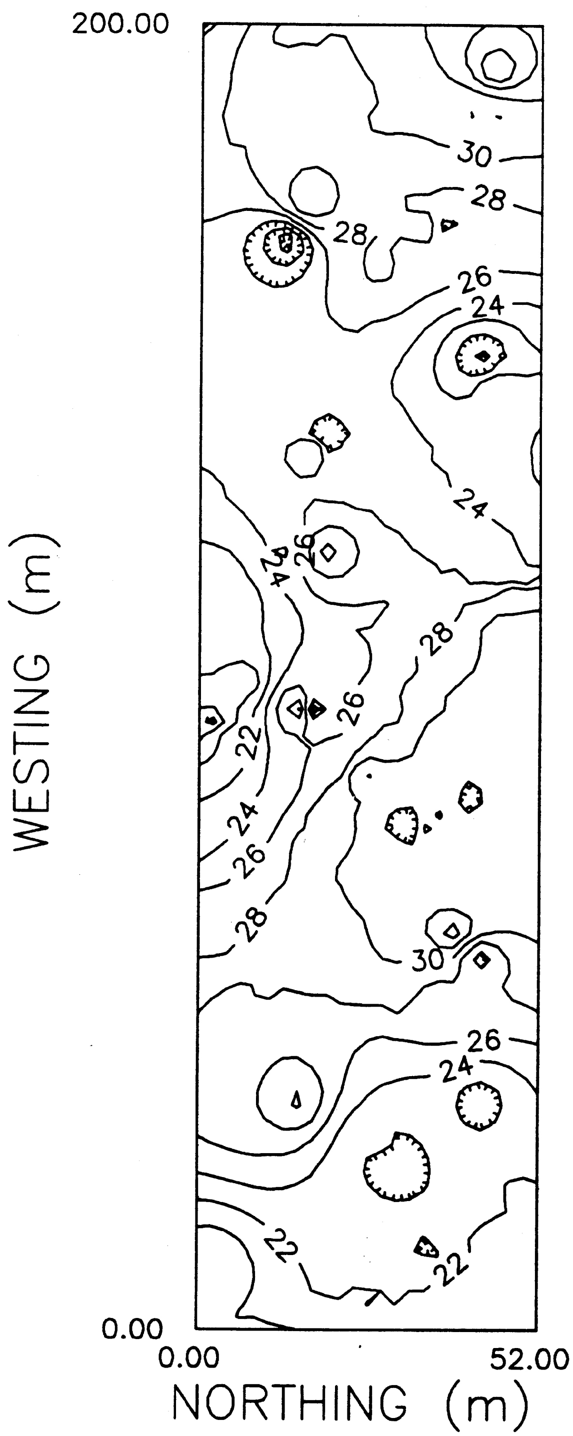

Figure 9-17. Contours of measured profile water content, day 92, Irrigation 2, Experiment 2. Contours are on a 2.0 cm increment.

Summary.

Of five variables examined only catch can depths and profile water content showed consistent spatial structure. The variogram for catch can depths was fit well by a spherical model and that for profile water contents with a linear model. The relative variograms for both these variables were stable over time and allowed the fitting of a single relative variogram model to each data set with the result that the relative variogram model could be scaled to provide a model for any particular day's data by simply multiplying the nugget and sill by the variable's mean squared. The usefulness of this was enhanced by the fact that the mean value could be reliably estimated by using a location identified as representative of the mean. These locations were identified by the time invariance analysis of Chapter 8. Thus a strong link was demonstrated between the existence of time invariance for a variable and the usefulness of kriging on that variable.

There was little discernible spatial structure for evaporation nor for the change in storage due to irrigation. The structure apparent for evaporation data from days 332 and 333, Experiment 3, is associated with an influx of warm air causing mean daily air temperature to rise 3 °C.

Data from some days seemed to show spatial structure for the midday temperature difference between dry and drying soil. However, data from many other days showed no spatial structure. There appeared to be some structure on the warm day 333 during Experiment 3. There was more structure apparent for the first 2 days after Irrigations 1 and 2 of Experiment 2. It was much warmer during the late March - early April time period of Experiment 2 than during the November - early December period of Experiment 3. Thus it appears that the appearence of spatial structure associated with soil surface temperature may be linked to ambient temperature or potential evapotranspiration.

The 'here today - gone tomorrow' nature of spatial structure associated with evaporation and surface temperature data makes questionable the utility of spatial variability analysis of these variables. In particular the idea, that cokriging using evaporation and temperature data could be used to reduce the number of ML samples needed to estimate evaporation, is shown to be questionable due to the lack of time invariant spatial structure.

Soil surface temperature is a rapidly changing environmental variable. In Chapter 3 it was shown that the surface temperature at midday could vary by up to 10 °C over the 30 to 40 minutes necessary to measure all 57 locations in the field. Changes in cloud cover and wind speed were associated with these temperature changes. Such rapid temperature variations render problematic the task of measuring the spatial structure of soil surface temperature.

The geostatistical approach was initially developed to study the spatial structure of ore grades, a variable that changes with the millenia. It is perhaps asking too much to apply these same techniques to labile environmental variables.